Eur. Phys. J. C 46, 407–412 (2006)
Digital Object Identifier (DOI) 10.1140/epjc/s2006-02505-y

THE EUROPEAN PHYSICAL JOURNAL C

One-loop corrections to the Drell–Yan process in SANC: the charged current case

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Received: 9 January 2006 /
Published online: 17 March 2006 –

Abstract. Radiative corrections to the charged current Drell–Yan processes are revisited. Complete one-loop electroweak corrections are calculated within the automatic SANC system. Electroweak scheme dependence and the choice of the factorization scale are discussed. Comparisons with earlier calculations are presented.

PACS. 13.85.Qk; 12.15.Lk

1 Introduction

Precision studies of the Drell–Yan process are vitally important for high energy hadronic colliders. This process provides information about weak interactions and contributes to the background to many of the searches for physics beyond the standard model. One-loop QED and electroweak (EW) radiative corrections (RC) to the Drell–Yan process at high energy hadronic colliders were calculated by several groups in the past; see [1–6] and references therein. Here we present the results for the corrections to the charged current Drell–Yan process, obtained within the automated system SANC [7, 8], and some comparisons with earlier calculations. Starting from the construction of helicity amplitudes and EW form factors, SANC performs a calculation of the process cross section and produces computer codes, which can be further used in the experimental data analysis.

2 Preliminaries and notation

Let us start with the partonic level, where we will consider interactions of *free* quarks (partons). The differential Born-level cross section of the process

$$\bar{d}(p_1) + u(p_2) \to l^+(p_4) + \nu_l(p_3) \tag{1}$$

in the center-of-mass system of the initial quarks reads

$$\frac{\mathrm{d}\hat{\sigma}_0}{\mathrm{d}\hat{\Omega}} = \frac{1}{4}\frac{1}{N_c}|V_{ud}|^2 \frac{G_{\mathrm{F}}^2 M_{\mathrm{W}}^2}{2\pi\hat{s}} \frac{\hat{u}^2}{(\hat{s}-M_{\mathrm{W}}^2)^2 + \Gamma_{\mathrm{W}}^2(\hat{s})M_{\mathrm{W}}^2},$$
$$\hat{s} = (p_1+p_2)^2\,, \qquad \hat{u} = (p_1-p_3)^2\,, \tag{2}$$

where $N_c = 3$ is the number of quark colors; V_{ud} is the relevant element of the CKM matrix; G_{F} is the Fermi coupling constant; and M_{W} and Γ_{W} are the mass and the width of the W-boson, respectively.

3 Radiative corrections at the partonic level

In order to obtain a more accurate description of the process, we should go beyond the Born approximation and take into account different sources of radiative corrections. Here we will consider only EW contributions to the corrections, while effects of higher-order QCD contributions (and *mixed* effects) are beyond the scope of our study.

As usually, we subdivide the EW RC into the virtual (loop) ones, the ones due to soft photon emission, and the ones due to hard photon emission. An auxiliary parameter $\bar{\omega}$ separates the soft and hard photonic contributions.

In the automated system [8], the virtual corrections are accessible via the menu chain **SANC** → **EW** → **Processes** → **4 legs** → **4f** → **Charged Current** → **f1 f1' → f f' (FF)**. The module, loaded at the end of this chain, computes on-line the scalar form factors of the partonic sub-process (1). The parallel module ... **f1 f1' → f f' (HA)** provides the relevant helicity amplitudes. For more details, see Sect. 2.5 of the SANC description [7] and the book [9].

The real photon emission process

$$\bar{d}(p_1) + u(p_2) \to l^+(p_4) + \nu_l(p_3) + \gamma(p_5) \tag{3}$$

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should be taken into account as well. Integration over the phase space in this case can be performed either (semi-)analytically or by means of a Monte Carlo integrator.

The first possibility is realized within the SANC environment. Now we have two branches there. The first contains the complete chain of analytical integrals over the hard photon phase space. It provides at the partonic level the double-differential distribution $\mathrm{d}^2\hat{\sigma}_{\mathrm{hard}}/(\mathrm{d}c\,\mathrm{d}\hat{s}')$ and the single differential distribution $\mathrm{d}\hat{\sigma}_{\mathrm{hard}}/\mathrm{d}c$, where $c = \cos\angle(p_2 p_4)$ and $\hat{s}' = (p_3 + p_4)^2$. The second branch provides the double-differential distribution $\mathrm{d}^2\hat{\sigma}_{\mathrm{hard}}/(\mathrm{d}c\,\mathrm{d}M_{\mathrm{x}}^2)$, where $M_{\mathrm{x}}^2 = 2p_3p_5$, which is directly related to the charged lepton energy in the center-of-mass system of the initial quarks:

$$\hat{E}_\mu = p_4^0 = \frac{\hat{s} + m_l^2 - M_{\mathrm{x}}^2}{2\sqrt{\hat{s}}}\,. \tag{4}$$

We also managed to obtain analytically the hard photon contribution as the single differential distribution $\mathrm{d}\hat{\sigma}_{\mathrm{hard}}/\mathrm{d}\hat{s}'$. In this case we can use a system of reference with the z-axis along the real photon momentum p_5. There the integration over three angular variables is rather easy and we even have a possibility of keeping all the light masses exactly. Below we give the expression without mass terms because of its simplicity:

$$\begin{aligned}\frac{\mathrm{d}\hat{\sigma}_{\mathrm{hard}}}{\mathrm{d}\hat{s}'} = \hat{\sigma}_0\,\frac{\alpha}{2\pi}\,\frac{1}{\hat{s}^2}\,\frac{1}{\hat{s}-\hat{s}'}\Biggl\{&\left[\hat{s}^2+\hat{s}'^2\right]\Biggl[Q_{\mathrm{l}}^2\\
&\times\left(\ln\frac{\hat{s}'}{m_l^2}-1\right)+\frac{\hat{s}'}{\hat{s}}\,\frac{(\hat{s}-M_{\mathrm{W}}^2)^2+\Gamma_{\mathrm{W}}^2M_{\mathrm{W}}^2}{(\hat{s}'-M_{\mathrm{W}}^2)^2+\Gamma_{\mathrm{W}}^2M_{\mathrm{W}}^2}\\
&\times\left[Q_{\mathrm{u}}^2\left(\ln\frac{\hat{s}}{m_u^2}-1\right)+Q_{\mathrm{d}}^2\left(\ln\frac{\hat{s}}{m_d^2}-1\right)\right]\Biggr]\\
&-\frac{2}{3}\left(\hat{s}^2+\hat{s}\hat{s}'+\hat{s}'^2\right)\Biggl[Q_{\mathrm{l}}^2\\
&+\frac{\hat{s}'}{\hat{s}}\,\frac{(\hat{s}-M_{\mathrm{W}}^2)^2+\Gamma_{\mathrm{W}}^2M_{\mathrm{W}}^2}{(\hat{s}'-M_{\mathrm{W}}^2)^2+\Gamma_{\mathrm{W}}^2M_{\mathrm{W}}^2}\Biggr]\\
&-\frac{1}{3}\left[\hat{s}'\left(\hat{s}+\hat{s}'\right)\right]Q_l\Bigl(4Q_u+5Q_d\Bigr)\\
&\times\frac{\left(M_{\mathrm{W}}^2-\hat{s}\right)\left(M_{\mathrm{W}}^2-\hat{s}'\right)+\Gamma_{\mathrm{W}}^2M_{\mathrm{W}}^2}{(\hat{s}'-M_{\mathrm{W}}^2)^2+\Gamma_{\mathrm{W}}^2M_{\mathrm{W}}^2}\Biggr\}\,,\end{aligned} \tag{5}$$

where Q_{l}, Q_{u}, and Q_{d} are the charges of the charged lepton, up-quark, and down-quark, respectively.

The differential distributions of the tree-level radiative process $\bar{d} + u \to l^+ + \nu_l + \gamma$ were compared with the corresponding distributions obtained by means of the CompHEP package [10]. Cross-section distributions in the cosine of the outgoing charged lepton (the muon is used) angle and in the lepton energy are considered. 20 bins are constructed for each of the distributions. Bins in the muon energy are

$$(n_{\mathrm{bin}} - 1) \times 5\,\mathrm{GeV} < E_\mu < n_{\mathrm{bin}} \times 5\,\mathrm{GeV}\,. \tag{6}$$

The cut on the muon energy ($E_\mu < 95\,\mathrm{GeV}$) is imposed in both distributions to avoid the region with soft photons, where CompHEP is not supposed to work well. The angular bins are

$$-1 + \frac{n_{\mathrm{bin}} - 1}{10} < c < -1 + \frac{n_{\mathrm{bin}}}{10}\,. \tag{7}$$

Table 1. Bin by bin comparison of differential distributions for the process $\bar{d} + u \to \mu^+ + \nu_\mu + \gamma$

	Bins in E_μ		Bins in c	
Bin	SANC	CompHEP	SANC	CompHEP
1	0.0006(1)	0.0006(1)	0.5867(1)	0.5869(2)
2	0.0010(1)	0.0010(1)	0.3538(1)	0.3537(1)
3	0.0023(1)	0.0023(1)	0.1857(1)	0.1858(1)
4	0.0452(1)	0.0451(1)	0.1111(1)	0.1111(1)
5	0.0569(1)	0.0569(1)	0.0740(1)	0.0741(1)
6	0.0549(1)	0.0549(1)	0.0541(1)	0.0541(1)
7	0.0546(1)	0.0546(1)	0.0427(1)	0.0428(1)
8	0.0563(1)	0.0563(1)	0.0360(1)	0.0361(1)
9	0.0603(1)	0.0603(1)	0.0321(1)	0.0321(1)
10	0.0667(1)	0.0666(1)	0.0298(1)	0.0297(1)
11	0.0755(1)	0.0755(1)	0.0287(1)	0.0287(1)
12	0.0868(1)	0.0867(1)	0.0284(1)	0.0285(1)
13	0.1008(1)	0.1008(1)	0.0286(1)	0.0287(1)
14	0.1175(1)	0.1175(1)	0.0292(1)	0.0292(1)
15	0.1372(1)	0.1372(1)	0.0299(1)	0.0299(1)
16	0.1605(1)	0.1605(1)	0.0302(1)	0.0301(1)
17	0.1881(1)	0.1881(1)	0.0294(1)	0.0293(1)
18	0.2227(1)	0.2227(1)	0.0263(1)	0.0263(1)
19	0.2739(1)	0.2740(2)	0.0187(1)	0.0187(1)
20	0.0	0.0	0.0059(1)	0.0059(1)

The $\alpha(M_{\mathrm{Z}})$ electroweak scheme (realized according to the CompHEP conventions) was used. An agreement was found, as can be seen from Table 1.

For the two choices of variables we have simple analytical expressions for the corresponding soft photon contributions. The infrared singularities in them are regularized by the auxiliary photon mass. The energy of a soft photon is limited from above by a cut in the integral either over $\hat{s}'$ or over M_{x}^2.

In order to have the possibility of imposing experimental cuts and event-selection procedures of any kind, we can use the Monte Carlo integration routine based on the Vegas algorithm [11]. In this case we perform a 4(6)-fold numerical integration to obtain the hard photon contribution to the partonic (hadronic) cross section. To obtain the total EW correction in this case we also add the contributions of the soft photon emission and the ones of the virtual loops. The cancelation of the dependence on the auxiliary parameter $\bar{\omega}$ in the sum is observed numerically.

Using the splitting of the W-boson propagators in the case of real photon emission off the virtual W, we separate the contributions of the initial-state radiation, the final-state one, and their interference in a gauge-invariant way [12]. The splitting is introduced by the following formula:

$$\frac{1}{\hat{s} - (M_{\mathrm{W}} - \mathrm{i}\Gamma_{\mathrm{W}})^2}\,\frac{1}{\hat{s}' - (M_{\mathrm{W}} - \mathrm{i}\Gamma_{\mathrm{W}})^2}$$

$$= \frac{1}{(\hat{s}-\hat{s}')}\left(\frac{1}{\hat{s}'-(M_{\mathrm{W}}-\mathrm{i}\Gamma_{\mathrm{W}})^2} - \frac{1}{\hat{s}-(M_{\mathrm{W}}-\mathrm{i}\Gamma_{\mathrm{W}})^2}\right). \tag{8}$$

In the center-of-mass system $(\hat{s}-\hat{s}') = 2p_5^0\sqrt{\hat{s}}$. The fixed W-width scheme is used here and in what follows.

In the course of calculations of the $\mathcal{O}(\alpha)$ corrections we met the so-called *on-shell* singularities, which appear in the form of $\ln(\hat{s}-M_{\mathrm{W}}^2+\mathrm{i}\epsilon)$. As was shown in detail in [3], they can be regularized by the W-width:

$$\ln(\hat{s}'-M_{\mathrm{W}}^2+\mathrm{i}\epsilon) \to \ln(\hat{s}'-M_{\mathrm{W}}^2+\mathrm{i}M_{\mathrm{W}}\Gamma_{\mathrm{W}}). \tag{9}$$

In the analytical formulae for radiative corrections one can find logarithms with quark and lepton mass singularities:

$$\ln\frac{\hat{s}}{m_{\mathrm{l}}^2}, \qquad \ln\frac{\hat{s}}{m_{\mathrm{u}}^2}, \qquad \ln\frac{\hat{s}}{m_{\mathrm{d}}^2}. \tag{10}$$

In the experimental set-up with calorimetric registration of the final-state charged particles (typical for electrons), the lepton mass singularity cancels out in the result for the correction to an observable cross section in accordance with the Kinoshita–Lee–Nauenberg theorem [13, 14]. But, if the experiment is measuring the energy of the charged lepton without summing it with the energies of accompanying collinear photons (typical for muons), the logarithms with the lepton mass singularity remain in the result and give a considerable numerical contribution. Re-summation of these logarithms in higher orders was discussed in [15–17].

3.1 Treatment of quark mass singularities

One-loop radiative corrections contain terms proportional to the logarithms of the quark masses, $\ln(\hat{s}/m_{\mathrm{u,d}}^2)$. They come from the initial-state radiation contributions including hard, soft, and virtual photon emission. Such initial-state mass singularities are well known, for instance, in the process of e^+e^- annihilation. But, in the case of hadron collisions these logarithms have been already *effectively* taken into account in the parton density functions (PDFs). In fact, in the procedure of PDF extraction from the experimental data, QED radiative corrections to the quark line have not been systematically subtracted. Therefore, the present PDFs effectively include not only the QCD evolution but also the QED one. Moreover, it is known that the leading logarithm behaviors of the QED and QCD Dokshitzer–Gribov–Lipatov–Altarelli–Parisi evolution of quark density functions are similar (proportional to each other). So, one obtains the evolution of the PDFs with an effective coupling constant

$$\alpha_s^{\mathrm{eff}} \approx \alpha_s + \frac{Q_i^2}{C_{\mathrm{F}}}\alpha, \tag{11}$$

where α_s is the strong coupling constant, α is the fine structure constant, Q_i is the quark charge, and C_{F} is the QCD color factor. The non-trivial difference between the QED evolution and the QCD one starts to appear in higher orders, and the corresponding numerical effect is small compared to the remaining QCD uncertainties in PDFs [18–21]. The best approach to the whole problem would be to re-analyze all the experimental deep inelastic scattering (DIS) data taking into account QED corrections to the quark line at least at the next-to-leading order. But, for the present moment we can limit ourselves with an application of a certain subtraction scheme to the QED part of the radiative corrections for the process under consideration. We will use here the $\overline{\mathrm{MS}}$ scheme [22]; the DIS scheme can be used as well. This allows us to avoid the double counting of the initial quark mass singularities contained in our result for the corrections to the free quark cross section and the ones contained in the corresponding PDF. The latter should also be taken in the same scheme with the same factorization scale.

In fact, using the initial condition for the non-singlet next-to-leading order QED quark structure function, which coincides with the QCD one with the trivial substitution $C_{\mathrm{F}}\alpha_s \to Q_i^2\alpha$, see [23], one obtains the following expression for the terms to be subtracted from the full calculation with massive quarks:

$$\delta^{\overline{\mathrm{MS}}} = \sum_{i=1,2} Q_i^2 \frac{\alpha}{2\pi}\int\limits_0^1 \mathrm{d}\xi_i \left[\frac{1+\xi_i^2}{1-\xi_i}\left(\ln\frac{M^2}{m_i^2} - 2\ln(1-\xi_i)-1\right)\right]_+ \hat{\sigma}_0(\xi_i), \tag{12}$$

where Q_i and m_i denote the charge and the mass of the given quark; M is the factorization scale; and $\hat{\sigma}_0(\xi_i)$ is the cross section at the partonic level with the reduced value of the quark momentum: $p_i \to \xi_i p_i$. The subtracted cross section with $\mathcal{O}(\alpha)$ corrections is given by

$$\hat{\sigma}_1^{\overline{\mathrm{MS}}} = \hat{\sigma}_1 - \delta^{\overline{\mathrm{MS}}}. \tag{13}$$

Then, it can be convoluted with PDFs as shown below in (16).

But, there is an alternative way to perform the subtraction. Really, to avoid the double counting of the quark mass singularities, we can leave them in the corrected cross section, but remove them from the PDFs:

$$\begin{aligned}\bar{q}(x,M^2) &= q(x,M^2) - \int_x^1 \frac{\mathrm{d}z}{z}\, q\left(\frac{x}{z},M^2\right)\frac{\alpha}{2\pi}Q_q^2 \\ &\quad\times\left[\frac{1+z^2}{1-z}\left\{\ln\left(\frac{M^2}{m_q^2}\right) - 2\ln(1-z)-1\right\}\right]_+ \\ &\equiv q(x,M^2) - \Delta q,\end{aligned} \tag{14}$$

where $q(x,M^2)$ can be taken directly from the existing PDFs in the $\overline{\mathrm{MS}}$ scheme (see [3] for the corresponding formula in the DIS scheme). It can be shown analytically (see e.g. [3]) that this procedure is equivalent to the subtraction from the cross section, and that it really removes (hides) the dependence on the quark masses. The advantage of the last approach is that it can be used regardless of the way of representing the partonic cross section: it can be kept even in the completely differential form.

The *natural* choices of the factorization scale are $M^2 = M_{\mathrm{W}}^2$ (when returning to the W-resonance is allowed by kinematic cuts) and $M^2 = \hat{s} = x_1 x_2 s$. Variations with respect to the choice should be studied.

In order to avoid the appearance of spurious higher-order terms for the case of subtraction from PDFs, we suggest to apply a procedure of *linearization*. Schematically, it can be represented as follows:

$$\begin{aligned}
&\bar{q}_1(x_1,M^2)\times\bar{q}_2(x_2,M^2)\times\hat{\sigma}_1 = [q_1(x_1,M^2)-\Delta q_1]\\
&\quad\times[q_2(x_2,M^2)-\Delta q_2]\times(\hat{\sigma}_{\mathrm{Born}}+\hat{\sigma}_\alpha)\\
&\quad\to q_1(x_1,M^2)\times q_2(x_2,M^2)\times\hat{\sigma}_{\mathrm{Born}}\\
&\quad+q_1(x_1,M^2)\times q_2(x_2,M^2)\times\hat{\sigma}_\alpha\\
&\quad-[q_1(x_1,M^2)\times\Delta q_2+q_2(x_2,M^2)\times\Delta q_1]\times\hat{\sigma}_{\mathrm{Born}}\,,
\end{aligned} \tag{15}$$

where $\hat{\sigma}_{\mathrm{Born}}$ and $\hat{\sigma}_\alpha$ denote the Born-level partonic cross section and the $\mathcal{O}(\alpha)$ RC contribution to it, respectively. Without the linearization procedure, terms with quark mass singularities would remain in the $\mathcal{O}\left(\alpha^2\right)$ contribution to the cross section.

4 Radiative corrections to hadronic processes

The double-differential cross section of the Drell–Yan process can be obtained from the convolution of the partonic cross section with the quark density functions:

$$\begin{aligned}
\frac{\mathrm{d}\sigma^{pp\to\mu^+\nu X}_{\mathrm{RC}}(s)}{\mathrm{d}c\;\mathrm{d}E_\mu} &= \sum_{q_1q_2}\int\limits_0^1\int\limits_0^1 \mathrm{d}x_1\;\mathrm{d}x_2\;\bar{q}_1(x_1,M^2)\\
&\times\bar{q}_2(x_2,M^2)\frac{\mathrm{d}^2\hat{\sigma}^{q_1q_2\to\mu^+\nu}(\hat{s})}{\mathrm{d}\hat{c}\,\mathrm{d}\hat{E}_\mu}\;\mathcal{J}\,\Theta(c,E_\mu)\,,
\end{aligned} \tag{16}$$

where the parton densities with *bars* mean the ones modified by the subtraction of the quark mass singularities and the step function $\Theta(c,E_\mu)$ defines the phase-space domain corresponding to the given event-selection procedure. The partonic cross section is taken in the center-of-mass reference frame of the initial quarks, where the cosine of the muon scattering angle, $\hat{c}$, and the muon energy, $\hat{E}_\mu$, are defined. The transformation into the observable variables c and E_μ involves the Jacobian:

$$\begin{aligned}
&\mathcal{J} = \frac{\partial\hat{c}}{\partial c}\frac{\partial\hat{E}_\mu}{\partial E_\mu} = \frac{4x_1x_2}{a^2}\sqrt{\frac{a^2(1+c)}{x_1[a+x_2(1+c)]}}\,,\\
&a = x_1+x_2-c(x_1-x_2),\quad \hat{c} = 1-(1-c)\frac{2x_1}{a}\,,\\
&\hat{s} = sx_1x_2,\qquad \hat{E}_\mu = \frac{\sqrt{\hat{s}}}{2}\,,\\
&\hat{E}_\mu = E_\mu\sqrt{\frac{1-c^2}{1-\hat{c}^2}}\,.
\end{aligned} \tag{17}$$

An analogous formula can be written for any other choice of a differential distribution as well as for the total cross section.

5 Numerical results and conclusions

For numerical evaluations we take the same set of input parameters as the one given by Eq. (4.1) of [5]. In Table 2 we present the results for the total cross section[1] of the process $u+\bar{d}\to\nu_l+l^+(+\gamma)$. For the Born-level cross section we completely (in all listed digits) agree with the numbers given in [5]. The third line shows radiative corrections in percent before the subtraction of quark mass singularities. These numbers were obtained directly from the SANC system for the GF EW scheme. Starting from the fourth line, we use the treatment of the EW scheme[2], which has been adopted [5]. The results for the radiative corrections with $\overline{\mathrm{MS}}$ subtraction (with factorization scale being equal to M_{W}) are also in fair agreement. The small deviations there can be due to details in the treatment of the EW scheme with respect to induced higher-order effects. Huge positive corrections in the case without subtraction of quark mass singularities above the W-peak are due to the initial-state radiation, which provides the radiative return to the W-resonance.

The effect of EW scheme dependence is illustrated by Table 3. Results for the total partonic cross section at

[1] The factor $|V_{ud}|^2$ has been dropped for the sake of comparison with [5].

[2] In the G_{F}' scheme we assigned the following one-loop value of the coupling constant standing at the photon vertices: $\alpha_{\mathrm{QED}}\approx 1/132.544$.

Table 2. The total lowest-order parton cross section $\hat{\sigma}_0$ in the G_{F} EW scheme and the corresponding relative one-loop correction δ

$\sqrt{\hat{s}}$/GeV	40	80	120	200	500	1000	2000
$\hat{\sigma}_0$/pb	2.646	7991.4	8.906	1.388	0.165	0.0396	0.00979
δ/%, full, G_{F}	−1.70	−7.62	89.9	125.3	155.9	166.9	173.0
δ/%, full, G_{F}'	−1.76	−7.87	92.9	129.6	161.2	172.5	178.9
δ/%, $\overline{\mathrm{MS}}(s)$, G_{F}'	0.56	2.48	−17.2	−16.0	−17.9	−23.2	−31.5
δ/%, $\overline{\mathrm{MS}}(M_{\mathrm{W}})$, G_{F}'	0.73	2.48	−12.9	−3.2	11.6	18.8	22.8
δ/% [5]	0.7	2.42	−12.9	−3.3	12	19	23

the Born and $\mathcal{O}(\alpha)$ levels are given for two EW schemes. At the Born level the 7.3% difference appears just due to the difference in the definition of EW constants in the G_{F} and in the $\alpha(0)$ schemes. As it should be, the difference between the corrected cross sections is less than the one at the Born level. But, still it is large and comparable with the ordered precision of the calculation. Certainly, usage of $\alpha(0)$ is not well motivated for the given energy range. And, the difference δ_1 gives only an upper estimate of the uncertainty due to the EW scheme dependence. In any case, we are going to perform further studies of this effect.

Table 4 represents the dependence of the hadronic Drell–Yan cross section on the values of the quark masses with and without the subtraction procedure. The conditions are as follows: the center-of-mass energy is 200 GeV; all events with the invariant mass of the neutrino and charged lepton pair above $\sqrt{40}$ GeV are accepted. σ_0 denotes the Born-level cross section obtained using the CTEQ4L set of PDFs [24]. σ_1, $\sigma_1^{\overline{\mathrm{MS}}(\sigma)}$, $\sigma_1^{\overline{\mathrm{MS}}(q)}$, and $\sigma_1^{\overline{\mathrm{MS}}(q)}$ (lin.) stand for the cross sections with one-loop EW RC included. The double counting of the quark mass singularities in σ_1 is not removed. The $\overline{\mathrm{MS}}$ procedure (13) is applied to the partonic cross section in the computation of $\sigma_1^{\overline{\mathrm{MS}}(\sigma)}$. Values of $\sigma_1^{\overline{\mathrm{MS}}(q)}$ and $\sigma_1^{\overline{\mathrm{MS}}(q)}$(lin.) are computed by convolution of the quark (parton) density function modified according to (14) with the full (including quark mass singularities) partonic cross section. The linearization procedure (15) was adopted for $\sigma_1^{\overline{\mathrm{MS}}(q)}$(lin.) in addition. One can see that the numerical effect of linearization for the given set-up is small (but visible). The two approaches to remove the double counting give very close results, as they should.

For an internal test of our calculations, a comparison of the results produced by our Monte Carlo (MC) and semi-analytical (SA) codes for the description of hard photon contributions was performed. The results are presented in Table 5, where the corresponding contributions to the proton–proton cross section at 14 TeV center-of-mass energy are given. The conditions and the input parameters were taken as the ones used in [25]:

Table 3. The total parton cross section in the G_{F} and $\alpha(0)$ EW schemes

$\sqrt{\hat{s}}$/GeV	40	80	120	200	500	1000	2000
$\hat{\sigma}_0$/pb, $[G_{\mathrm{F}}]$	2.646	7991.4	8.906	1.388	0.165	0.0396	0.00979
$\hat{\sigma}_0$/pb, $[\alpha(0)]$	2.454	7410.2	8.258	1.287	0.153	0.0368	0.00908
δ_0/% (diff)	−7.3	−7.3	−7.3	−7.3	−7.3	−7.3	−7.3
$\hat{\sigma}_1$/pb, $\overline{\mathrm{MS}}$ (M_{W}), $[G_{\mathrm{F}}]$	2.665	8183.2	7.796	1.345	0.183	0.0467	0.01195
$\hat{\sigma}_1$/pb, $\overline{\mathrm{MS}}$ (M_{W}), $[\alpha(0)]$	2.617	8029.5	7.721	1.324	0.179	0.0455	0.01162
δ_1/% (diff)	−1.8	−2.0	−0.5	−1.5	−2.6	−3.1	−3.3

Table 4. The tree level and corrected hadronic Drell–Yan cross section for different values of the light quark masses

	σ_0 [pb]	σ_1	$\sigma_1^{\overline{\mathrm{MS}}(\sigma)}$	$\sigma_1^{\overline{\mathrm{MS}}(q)}$	$\sigma_1^{\overline{\mathrm{MS}}(q)}$(lin.)
$m_u = m_d = 4.85$ MeV	2.5577(1)	2.4795(3)	2.5724(3)	2.5704(3)	2.5729(3)
$m_u = m_d = 48.5$ MeV	2.5577(1)	2.4992(3)	2.5724(3)	2.5713(3)	2.5727(3)
$m_u = m_d = 485$ MeV	2.5577(1)	2.5190(3)	2.5724(3)	2.5719(3)	2.5726(3)

Table 5. The hadronic Drell–Yan cross section in the Born approximation and the hard photon contributions to it for different values of the cut on the muon transverse momentum

p_T [GeV]	> 25	> 50	> 100	> 200	> 500	> 1000
σ_{Born} [pb]	2112.22(2)	13.1507(2)	0.94506(1)	0.115106(1)	0.00548132(6)	0.000262108(3)
			$\bar{\omega} = 0.01$ GeV			
$\delta\sigma_{\mathrm{hard}}^{\mathrm{MC}}$ [%]	27.52(2)	34.02(2)	43.88(2)	51.22(2)	59.67(2)	65.22(2)
$\delta\sigma_{\mathrm{hard}}^{\mathrm{SA}}$ [%]	27.54(2)	34.02(2)	43.87(2)	51.21(2)	59.68(2)	65.22(2)
$\delta\sigma_{\mathrm{tot}}^{\mathrm{MC}}$ [%]	−2.69(2)	−3.83(2)	−6.67(2)	−11.77(2)	−22.28(2)	−33.36(2)
$\delta\sigma_{\mathrm{tot}}^{\mathrm{SA}}$ [%]	−2.68(2)	−3.83(2)	−6.68(2)	−11.78(2)	−22.27(2)	−33.36(2)
			$\bar{\omega} = 0.001$ GeV			
$\delta\sigma_{\mathrm{hard}}^{\mathrm{MC}}$ [%]	36.85(2)	44.35(2)	55.70(2)	64.25(2)	74.14(2)	80.72(2)
$\delta\sigma_{\mathrm{hard}}^{\mathrm{SA}}$ [%]	36.88(2)	44.37(2)	55.70(2)	64.26(2)	74.15(2)	80.72(2)
$\delta\sigma_{\mathrm{tot}}^{\mathrm{MC}}$ [%]	−2.70(2)	−3.85(2)	−6.69(2)	−11.76(2)	−22.29(2)	−33.35(2)
$\delta\sigma_{\mathrm{tot}}^{\mathrm{SA}}$ [%]	−2.67(2)	−3.83(2)	−6.69(2)	−11.76(2)	−22.28(2)	−33.35(2)

$$
\begin{array}{llll}
G_{\mathrm{F}} & =1.16637\times 10^{5}\,\mathrm{GeV}^{-2}\,, & & \\
\alpha(0) & =1/137.03599911, & \alpha_s & =0.1187\,, \\
M_{\mathrm{W}} & =80.425\,\mathrm{GeV}\,, & \Gamma_{\mathrm{W}} & =2.124\,\mathrm{GeV}\,, \\
M_{\mathrm{Z}} & =91.1867\,\mathrm{GeV}\,, & \Gamma_{\mathrm{Z}} & =2.4952\,\mathrm{GeV}\,, \\
M_{\mathrm{H}} & =150\,\mathrm{GeV}\,, & m_t & =174.17\,\mathrm{GeV}\,, \\
m_u & =m_d=66\,\mathrm{MeV}\,, & m_c & =1.55\,\mathrm{GeV}\,, \\
m_s & =150\,\mathrm{MeV}\,, & m_b & =4.5\,\mathrm{GeV}\,, \\
|V_{ud}| & =|V_{cs}|=0.975\,, & |V_{us}| & =|V_{cd}|=0.222\,.
\end{array}
$$

The MRST204QED set [21] of PDFs and the G_{F} EW scheme were used. Six values for the cut on the muon transverse momentum, P_T, are considered. The cut on the muon rapidity is $|\eta_l|<1.2$. The cut on the missing momentum was not imposed, since it cannot be realized in the semi-analytical branch. We also show there the values of the total one-loop EW correction, $\delta\sigma^{\mathrm{MC,SA}}_{\mathrm{tot}}$. Table 5 shows results for two values of the soft–hard photon separator, $\bar{\omega}$, and justifies the independence of the total correction on it within the accuracy achieved. The separator is defined in the center-of-mass reference frame of the colliding quarks (partons). We stress that having a semi-analytical branch of calculations served us as a benchmark and helped a lot to adjust the Monte Carlo code.

In this way with the help of the automated SANC system we calculated the complete one-loop radiative corrections to the charged current Drell–Yan cross section. Our results at the partonic level are in good agreement with the ones published earlier in [5]. The corresponding computer codes in analytical (`FORM`) and numerical (`FORTRAN`) formats are available from SANC [8]. They can be used as a part of a more general computer program (like a Monte Carlo event generator) to describe the Drell–Yan process in realistic conditions. Further comparison at the hadronic level with analogous calculations of other groups is in progress [25].

Acknowledgements. We are grateful to C.C. Calame, S. Dittmaier, S. Jadach, M. Krämer, G. Montagna, O. Nicrosini, W. Placzek, A. Vicini, and Z. Was for discussions. Three of us (D.B., L.K., and G.N.) are indebted to the directorate of IFJ (Cracow, Poland) for hospitality extended to them in April–May 2005, when a part of this study was completed. This work was supported by the INTAS grant 03-51-4007. One of us (A.A.) acknowledges support by the grant of the President of the RF (Scientific Schools 2027.2003.2) and the RFBR grant 04-02-17192.

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